#2) $\Rightarrow k\dfrac{\partial^2 u}{\partial x^2} - u = \dfrac{\partial u}{\partial t} \rightarrow$ let $u = XT$

$\Rightarrow kX''T - XT = XT' \rightarrow$ divide by XT

$$\Rightarrow \frac{kX''\cancel{T}}{X\cancel{T}} - \frac{\cancel{XT}}{\cancel{XT}} = \frac{\cancel{X}T'}{\cancel{X}T}$$

$$\Rightarrow \frac{kX''}{X} - 1 = \frac{T'}{T} = -\lambda$$ ← constant

<u>Therefore</u>

$$\frac{T'}{T} = -\lambda \Rightarrow T' = -\lambda T \Rightarrow T' + \lambda T = 0$$

$$\Rightarrow (D+\lambda)T = 0 \Rightarrow D = -\lambda \Rightarrow \boxed{T = c_1 e^{-\lambda t}}$$

done with T!!

<u>Now Let's Do X</u>

$$\frac{kX''}{X} - 1 = -\lambda \Rightarrow kX'' - X = -\lambda X$$

$$\Rightarrow kX'' - X + \lambda X = 0 \Rightarrow kX'' + (\lambda - 1)X = 0$$

$$\Rightarrow X'' + \left(\frac{\lambda - 1}{k}\right)X = 0 \Rightarrow D^2 + \frac{\lambda - 1}{k} = 0$$

$$\Rightarrow D^2 = -\frac{(\lambda - 1)}{k} \Rightarrow D = \pm\sqrt{-\frac{(\lambda - 1)}{k}}$$

Continues

$$D = \pm\sqrt{-\frac{(\lambda-1)}{k}}$$

① if $\frac{(\lambda-1)}{k} = 0 \Rightarrow D = 0, 0$

$\therefore X = a_1 e^{0x} + a_2 x e^{0x}$

$\Rightarrow \boxed{X = a_1 + a_2 x}$

$\Rightarrow \frac{\lambda-1}{k} = 0 \Rightarrow \lambda = 1 \Rightarrow T = c_1 e^{-\lambda t} = c_1 e^{-t}$

$\therefore u = XT = c_1 e^{-t}(a_1 + a_2 x)$

combine constants

i.e. $c_1 a_1 = c_1$, $c_1 a_2 = c_2$

$\therefore \boxed{u = e^{-t}(c_1 + c_2 x)}$

Problem 3

Here is a start

Let $u = XT$

$$\therefore \quad \frac{a^2 X'' \cancel{T}}{\alpha^2 \cancel{X} \cancel{T}} = \frac{\cancel{X} T''}{\alpha^2 \cancel{X} T} + \frac{2k \cancel{X} T'}{\alpha^2 \cancel{X} T}$$

$$\Rightarrow \quad \frac{X''}{X} = \frac{T''}{\alpha^2 T} + \frac{2kT'}{\alpha^2 T} = -\lambda$$

Solve for T

$$\frac{T''}{\alpha^2 T} + \frac{2kT'}{\alpha^2 T} = -\lambda \Rightarrow T'' + 2kT' = -\lambda\alpha^2 T$$

$$\Rightarrow T'' + 2kT' + \lambda\alpha^2 T = 0$$

$$\Rightarrow D^2 + 2kD + \lambda\alpha^2 = 0 \qquad D = \frac{-2k \pm \sqrt{4k^2 - 4\lambda\alpha^2}}{2}$$

$$\Rightarrow D = \frac{-2k \pm 2\sqrt{k^2 - \lambda\alpha^2}}{2} = \boxed{-k \pm \sqrt{k^2 - \lambda\alpha^2} = D_T}$$

this is ~~the~~ D
for $T(t)$

② If: $\frac{\lambda-1}{k} = -\alpha^2 \Rightarrow D = \pm\sqrt{\alpha^2} = \pm\alpha$

↑ real #

$$\Rightarrow X = c_1 e^{\alpha x} + c_2 e^{\alpha x}$$

$$\therefore \frac{\lambda-1}{k} = -\alpha^2 \Rightarrow \lambda - 1 = -k\alpha^2$$

$$\Rightarrow \boxed{\lambda = 1 - k\alpha^2}$$

$$\therefore T = c_1 e^{-\lambda t} = c_1 e^{(k\alpha^2-1)t}$$

$$\therefore u = XT = c_1 e^{(k\alpha^2-1)t}\left(c_1 e^{\alpha x} + c_2 e^{\alpha x}\right)$$

combine constants

& finish

③ If $\frac{\lambda-1}{k} = \alpha^2$

(can you finish...

$\Rightarrow$ Solve X

$$\frac{X''}{X} = -\lambda \Rightarrow X'' + \lambda X = 0 \Rightarrow D^2 + \lambda = 0$$

$$\therefore \boxed{D_x = \pm\sqrt{-\lambda}}$$

This is D for X(x)

Scenario 1

$$\lambda = 0 \Rightarrow D_x = 0, 0, \Rightarrow \boxed{X = C_1 + C_2 X}$$

$$D_T = -k \pm \sqrt{k^2} = -k \pm k = 0, -2k$$

$$\therefore T = C_3 + C_4 e^{-2kt}$$

$$\therefore u = (C_1 + C_2 x)(C_3 + C_4 e^{-2kt})$$

Scenario 2

$$\lambda = -\alpha^2 \Rightarrow D_x = \pm\alpha, \quad D_T = -k \pm \sqrt{k^2 + \alpha^4}$$

$$\therefore X = C_1 e^{\alpha x} + C_2 e^{-\alpha x}$$

$$T = C_3 e^{(-k + \sqrt{k^2 + \alpha^2})t} + C_4 e^{(-k - \sqrt{k^2 + \alpha^4})}$$

$$\Rightarrow u = XT = (* \; *)(* \; *)$$

can you do the Rest